

F1 NAGD Revision Worksheet

1. Number

For help with all your revision:
www.mymaths.co.uk
username: jersey
password: factorise

Indices

Do you know the Index Laws?

1. The multiplication rule: $y^a \times y^b = y^{a+b}$
(e.g. $z^2 \times z^3 = z^{2+3} = z^5$)
2. The division rule: $y^a \div y^b = y^{a-b}$
(e.g. $10^9 \div 10^3 = 10^{9-3} = 10^6$)
3. The brackets rule: $(y^a)^b = y^{ab}$
(e.g. $(p^4)^2 = p^{4 \times 2} = p^8$)
4. The power of zero: $y^0 = 1$
5. The power of one: $y^1 = y$
6. Negative powers: $y^{-a} = 1/y^a$
(e.g. $4^{-2} = 1/4^2 = 1/16$)
7. Fractional powers: $y^{a/b} = \sqrt[b]{y^a}$
(e.g. $49^{1/2} = \sqrt{49} = 7$
or $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$)

Can you use the index laws to evaluate these...?

- $\frac{1}{2^{-4}}$
- $16^{1/2}$
- $8^{1/3}$
- $9^{3/2}$
- $64^{-2/3}$
- 187^0



Surds

Can you understand and calculate with surds?

1. $2 + \sqrt{3}$ is a surd. It has a whole number part and a square root (that is irrational)
2. Calculating:
 - $\sqrt{3} \times \sqrt{3} = 3$
 - $\sqrt{3} \times \sqrt{2} = \sqrt{6}$
 - $(2 + \sqrt{3}) \times \sqrt{3} = 2\sqrt{3}$
 - $(2 + \sqrt{3})(1 + \sqrt{3}) = 2 + 3\sqrt{3} + 3 = 5 + 3\sqrt{3}$
3. Simplifying:
 - $\sqrt{75} = \sqrt{(25 \times 3)} = 5\sqrt{3}$ (A square number)
4. Rationalising: Making the denominator a whole number
 - $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ (multiply top and bottom by $\sqrt{2}$)
 - $\frac{1 + \sqrt{3}}{2 + \sqrt{3}} = \frac{(1 + \sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = \frac{(1 + \sqrt{3})(2 - \sqrt{3})}{1}$
(multiply top and bottom by the conjugate)

Can you use your knowledge of surds to simplify these...?

1. $\sqrt{3} \times \sqrt{12}$
2. $\sqrt{5} \times \sqrt{2}$
3. $\frac{\sqrt{30}}{\sqrt{5}}$
4. $\frac{1}{\sqrt{2}}$
5. $\sqrt{18}$
6. $(\sqrt{3} + 1)(2\sqrt{3} + 5)$
7. $\frac{2}{\sqrt{3}}$
8. $\frac{3}{1 + \sqrt{2}}$
9. $\sqrt{3} + \sqrt{12} + \sqrt{48}$
10. $\frac{2 + 3\sqrt{5}}{1 - \sqrt{5}}$

2. Algebra

Quadratics

Watch the negatives!

Can you factorise these quadratic expressions?

$$6x^2 + 7x + 2$$

$$9x^2 - 25$$

$$3x^2 + 14x - 5$$

$$6x^2 - 13x + 6$$

Do you know how to **complete the square** to solve a quadratic?

Write the equation as ☺ ☹ ☹

$$(ax + b)^2 = c,$$

Then find the square root of both sides and simplify

Try these:

$$1. (x - 6)^2 = 2$$

$$2. x^2 + 2x - 6 = 0$$

$$3. 9x^2 - 12x - 1 = 0$$

Do you know how to use the quadratic formula?

If

$$ax^2 + bx + c = 0$$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

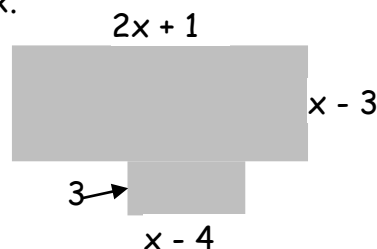
Use the formula to solve these: ☺ ☹ ☹

$$\bullet x^2 + 5x - 4 = 0$$

$$\bullet 3x^2 - x - 1$$

The area of the shape is 3 m^2 .

Write an equation for the area and work out x .



Algebraic Fractions

Can you simplify fractions?

$$\frac{6xy}{12y^2}$$

$$\frac{15m^2n}{10mn^2}$$

$$\frac{2t + 6}{t^2 + 6t + 9}$$

☺ ☹ ☹

Can you add fractions?

$$\frac{5}{x-1} + \frac{4}{x+5}$$

☺ ☹ ☹

Can you subtract fractions?

$$\frac{x-2}{5} - \frac{3x}{10}$$

☺ ☹ ☹

Can you multiply fractions?

$$\frac{x^3}{7xy} \times \frac{21y^2}{x^4}$$

☺ ☹ ☹

Can you divide fractions?

$$\frac{40}{x^3y} \div \frac{8}{xy^4}$$

☺ ☹ ☹

Can you use function notation?

$$f(x) = 5x + 5$$

$$g(x) = 9x - 4$$

$$h(x) = x^2 - 3x - 4$$

1) Find the value of each function if $x = 4$

2) Find the value of x if:

a) $f(x) = 65$

b) $g(x) = -22$

3) Find the value of x if $f(x) = 2g(x)$



Can you find composite functions?

$$f(x) = x^2 + 6$$

$$g(x) = 2x - 1$$

a) Find $fg(x)$

b) Find $gf(x)$

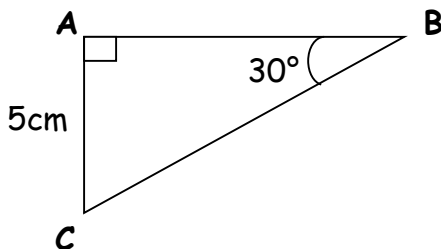
c) Find $gg(x)$



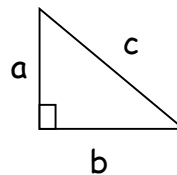
3. Geometry

Pythagoras and Trigonometry

Can you find the lengths of AB and BC?



Pythagoras' theorem states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

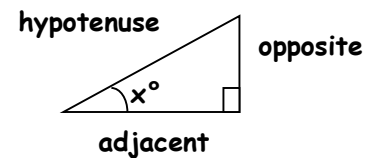


$$c^2 = a^2 + b^2$$

Trigonometry ratios

$$\tan x^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cos x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

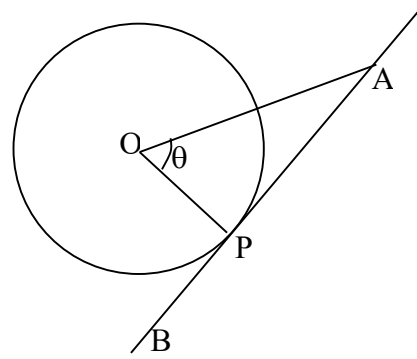
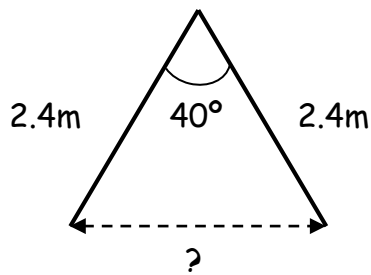


$$\sin x^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$

Remember: SOH CAH TOA



A step ladder has both legs 2.4 metres long. When open the angle at the top of the ladder is 40° . How far apart are the feet of the ladder?



The line extending through AB is a tangent to a circle with centre O and radius 5cm . The line AB intersects with the circle at P . The length of AP is 8cm .

Calculate the size of the angle labelled θ correct to 2 significant figures.

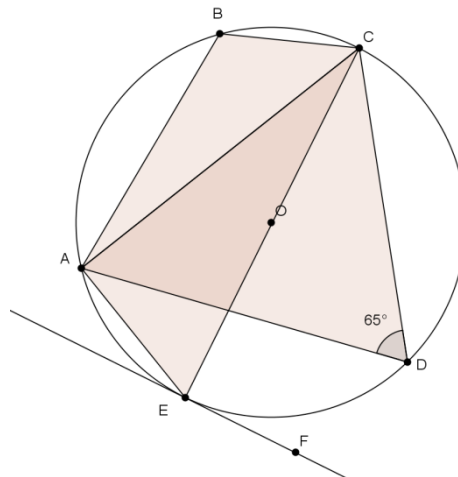


Angle theorems

Do you know all these?

Can you prove them?

- The angle in a semicircle is 90 degrees
- The angle at the centre is twice the angle at the circumference
- Angles subtended from the same chord are equal
- Opposite angles in a cyclic quadrilateral add up to 180
- The alternate segment theorem
- Tangents and radii meet at 90
- Two tangents from the same point are equal in length
- The perpendicular from the centre of a circle to a chord bisects the chord



O is the centre of the circle.

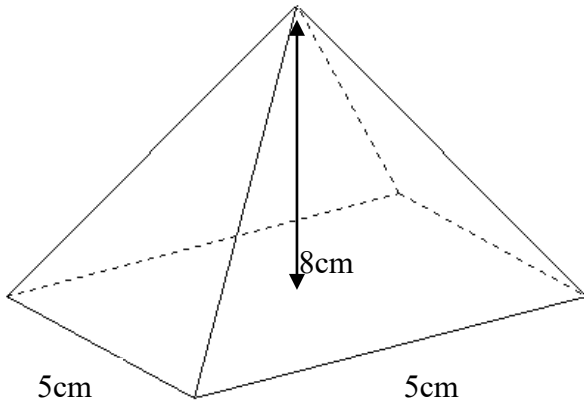
Angle $ADC = 65^\circ$.

Work out each angle and state which fact you are using. (You will have to draw some extra lines)

CEF, AOC, ABC, CAE, AEC

Draw a diagram to show each angle theorem.

3D Pythagoras and Trig



The square based pyramid has base lengths 5cm and height 8cm.

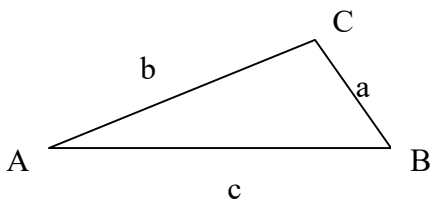
What is the length of a sloping edge?

What angle does a sloping edge make with the base?

What is the area of a sloping face?

Non-right angle triangles

Have you learned the following formulae?

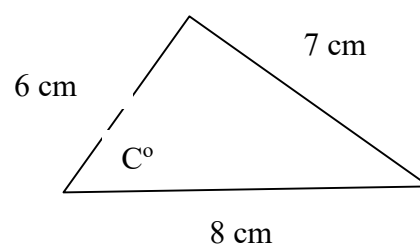
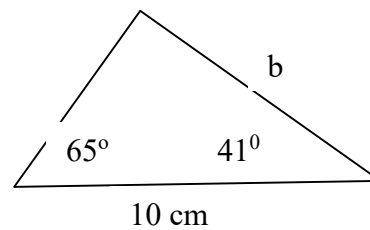
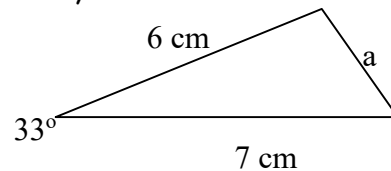


Area of a triangle = $\frac{1}{2} ab \sin C$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Work out the lengths and angles shown by letters, and each area :



4. Data Handling

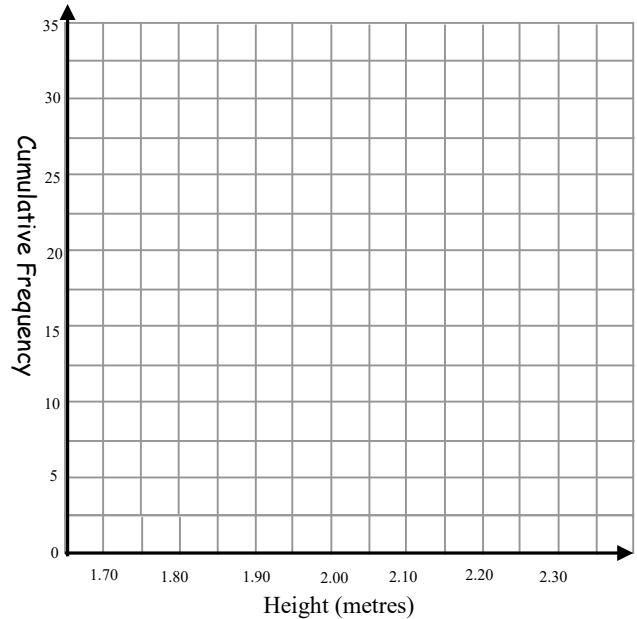
1. Can you complete a cumulative frequency table? 😊 😐 😞

Below are the heights (in metres) of a sample of 35 professional basketball players.

Height, h (metres)	Frequency	Cumulative Frequency
$1.70 < h \leq 1.80$	3	
$1.80 < h \leq 1.85$	4	
$1.85 < h \leq 1.90$	8	
$1.90 < h \leq 1.95$	8	
$1.95 < h \leq 2.00$	5	
$2.00 < h \leq 2.10$	4	
$2.10 < h \leq 2.25$	3	

2. Can you draw a cumulative frequency curve? 😊 😐 😞

Use your cumulative frequency table to draw a cumulative frequency curve.

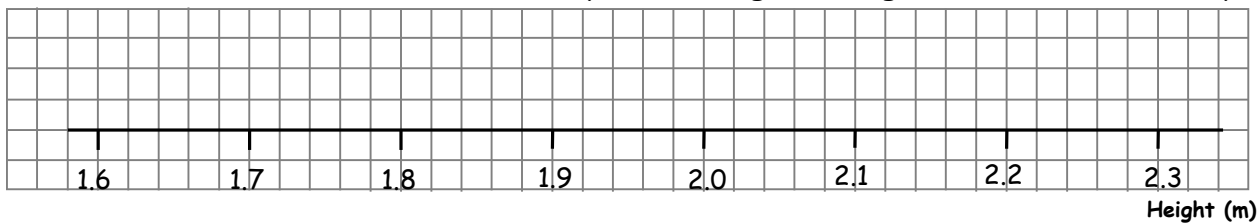


3. Can you use your cumulative frequency curve to find the Quartiles and the Interquartile Range (IQR)? 😊 😐 😞

Median	Lower Quartile	Upper Quartile	Interquartile Range

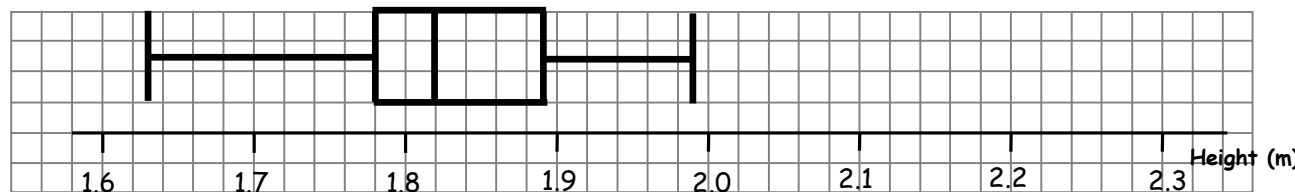
4. Can you draw a box plot (aka "box-and-whisker diagram")? 😊 😐 😞

Use the information above to draw a box plot showing the heights of the basketball players.



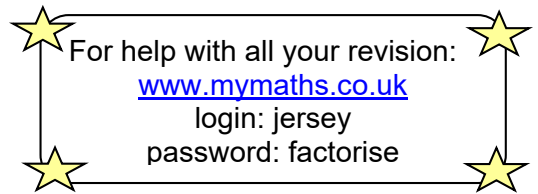
5. Can you compare two sets of data by reference to their box plots? 😊 😐 😞

Here is a box plot showing the heights of a sample of professional football players.



What comparisons can you make between the basketball players and the football players?

F2 Revision Worksheet



Number (FNum2)

1. Can you calculate simple and compound interest ?



a) Mary earns 5% simple interest per year on the £2000 she has in Trustbank. Grace receives 5% compound interest per year on the £2000 she has in Growbank. If both of them leave their savings untouched, calculate the difference between their savings at the end of two years.

Mary:..... Grace:..... Difference:.....

2. Can you calculate repeated percentage changes using the power key on your calculator ?



$$\text{Final amount} = \text{Starting amount} \times \text{Rate}^{\text{(time periods)}}$$

Eg 1: Find the value after 4 years of an investment of £4000 at a compound interest rate of 6%.

$$\begin{aligned} \text{New amount} &= 4000 \times 1.06^4 \\ &= 4000 \times 1.2624... \\ &= \pounds 5049.91 \end{aligned}$$

Eg 2: Find the value after 6 years of an £8000 car depreciating at a rate of 5% per year.

$$\begin{aligned} \text{New amount} &= 8000 \times 0.95^6 \\ &= 8000 \times 0.7350... \\ &= \pounds 5880.74 \end{aligned}$$

The single number the starting amount can be multiplied by to get the answer

a) An estate agent tells George that the houses in his area are likely to increase in value by 7% per year for the next 10 years. If George's house is worth £250,000 now, how much is it likely be worth in 10 years ?

Answer

b) A population of bats is forecast to decrease by 2% every year. There are currently 500 bats in the population, how many will there be in 8 years time ?

Answer

3. Can you work with general exponential growth and decay ?



a) The population of rabbits increases exponentially. In year 1 there were 400 rabbits. By year 3 there were 529 rabbits. How many rabbits were there by year 5?

b) The bacteria in a petri dish after one hour is 10 per cm^2 . After 5 hours it is 1690 per cm^2 . How many bacteria (per cm^2) will there be in the dish after 9 hours?

c) In 2010 in Tidmouth, 40,000 people were unemployed. By 2012 this had decreased to 39,601. Assuming the unemployment rate is decreasing exponentially, how many people would you expect to be unemployed in Tidmouth by 2020?

4. Can you use the product rule for counting ?



Remember: If there are m ways of doing one thing, and n ways of doing another, then the total number of ways the two things can be done is $m \times n$ (think of a sample space diagram from probability).

a) i) There are 17 men and 26 women in a choir.
The choir is going to sing at a concert.

One of the men and one of the women are going to be chosen to make a pair to sing the first song.

Work out the number of different pairs that can be chosen.

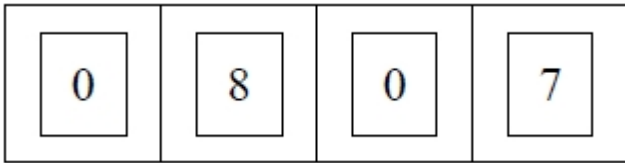
ii) Two of the men are to be chosen to make a pair to sing the second song.

Ben thinks the number of different pairs that can be chosen is 136
Mark thinks the number of different pairs that can be chosen is 272

Who is correct, Ben or Mark?
Give a reason for your answer.

b) Pavel has a combination lock.

Pavel has to set each part of the lock to a digit between 0 and 9 inclusive. One possible way to do this is shown in the diagram.



(i) How many different ways can Pavel do this?

Pavel decides that the 1st and 3rd digits will be odd numbers and that the 2nd and 4th digits will be even numbers greater than 0.

(b) How many different ways are possible now?

Algebra (FAlg2)

1. Can you use substitution to solve simultaneous equations ? ☺ ☹ ⊗

a)
$$\begin{aligned} 3x - 2y &= 0 \\ 2x + y &= 7 \end{aligned}$$

$x = \dots\dots\dots y = \dots\dots\dots$

b)
$$\begin{aligned} p + 2q &= 8 \\ 2p + 3q &= 14 \end{aligned}$$

$x = \dots\dots\dots y = \dots\dots\dots$

c)
$$\begin{aligned} c + 2(d - 6) &= 0 \\ 3c + 4d &= 30 \end{aligned}$$

$x = \dots\dots\dots y = \dots\dots\dots$

d)
$$\begin{aligned} 5f - g - 3 &= 8 \\ 4f + 3g &= -5 \end{aligned}$$

$x = \dots\dots\dots y = \dots\dots\dots$

2. Can you solve simultaneous equations using substitution where the graph of one is a straight line and the other is curved ?



Solve the following pairs of simultaneous equations by substitution

<p>a) $y = x^2 - 2x$ $y = x + 4$</p> <p>$x = \dots\dots\dots y = \dots\dots\dots$ or $x = \dots\dots\dots y = \dots\dots\dots$</p>	<p>b) $y = 7x - 8$ $y = x^2 - x + 7$</p> <p>$x = \dots\dots\dots y = \dots\dots\dots$ or $x = \dots\dots\dots y = \dots\dots\dots$</p>	<p>c) $y = 4x - 8$ $y^2 = 16x$</p> <p>$x = \dots\dots\dots y = \dots\dots\dots$ or $x = \dots\dots\dots y = \dots\dots\dots$</p>
<p>d) $y = x + 1$ $x^2 + y^2 = 9$</p> <p>$x = \dots\dots\dots y = \dots\dots\dots$ or $x = \dots\dots\dots y = \dots\dots\dots$</p>	<p>e) $y - x = 8$ $x^2 + y^2 = 34$</p> <p>$x = \dots\dots\dots y = \dots\dots\dots$ or $x = \dots\dots\dots y = \dots\dots\dots$</p>	<p>f) Explain why there are no solutions to this pair of simultaneous equations:</p> <p>$x^2 + y^2 = 1$ $y = x + 10$</p>

3. Can you solve simultaneous equations from graphs where the graph of one is a straight line and the other is curved ? 😊 😐 😞

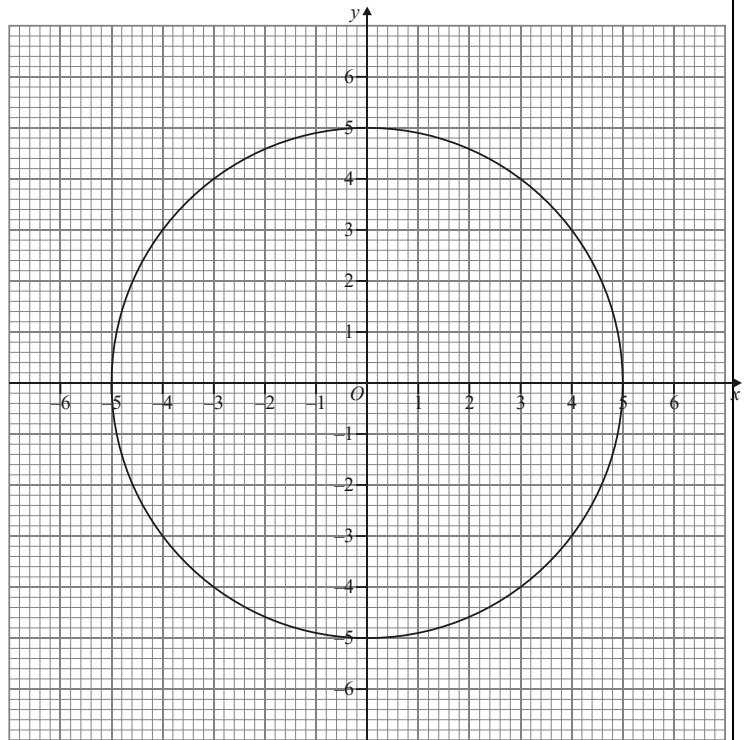
The diagram shows a circle of radius 5 cm, centre the origin.

Draw a suitable straight line on the diagram to find estimates of the solutions to the pair of equations below:

$$\begin{aligned} x^2 + y^2 &= 25 \\ y &= 2x + 1 \end{aligned}$$

$x = \dots\dots\dots, y = \dots\dots\dots$

$x = \dots\dots\dots, y = \dots\dots\dots$



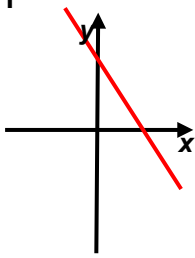
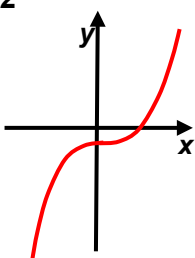
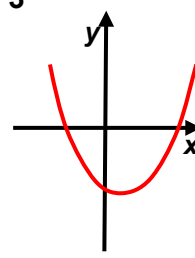
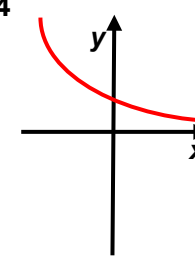
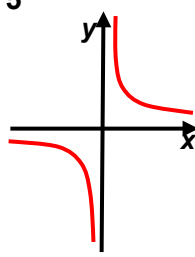
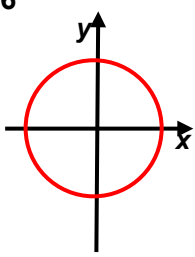
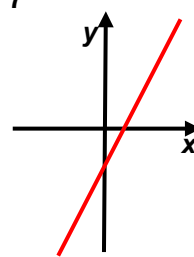
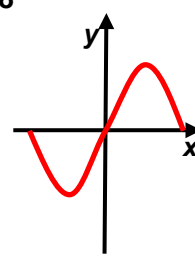
4. Can you use iterative methods to approximate solutions ? 😊 😐 😞

1 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

5. Can you match these graphs with their equations ? 😊 😐 😞

<p>1</p>  <p>.....</p>	<p>2</p>  <p>.....</p>	<p>3</p>  <p>.....</p>	<p>4</p>  <p>.....</p>
<p>5</p>  <p>.....</p>	<p>6</p>  <p>.....</p>	<p>7</p>  <p>.....</p>	<p>8</p>  <p>.....</p>

$y = x^2 - 3x - 5$ $y = -x^2 + 6x$ $y = \frac{2}{x}$ $y = -2x + 4$
 $y = x^3 - 5$ $y = 3x - 2$ $y = 0.5^x$
 $y = \sin x$

6. Can you draw up a table of values for each of the equations above and plot their graphs accurately ? 😊 😐 😞

Draw the graphs above accurately on separate pieces of **graph paper**.
 Start by finding the corresponding values for y when x = 0, 1, 2, 3, -1, -2 etc

Geometry (FGeom2)

Learn these formulas!

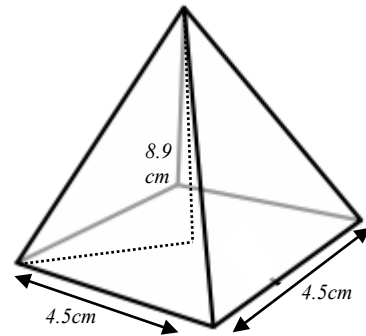
Formulas

Pyramid volume	$\frac{1}{3} \text{BaseArea} \times h$	Sphere volume	$\frac{4}{3} \pi r^3$
Cone volume	$\frac{1}{3} \pi r^2 \times h$	Sphere surface area	$4\pi r^2$
Cone surface area	$\pi r l + \pi r^2$	Circle segment area	$\frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \text{Sin} \theta$

1. Can you calculate the volume of a pyramid ?



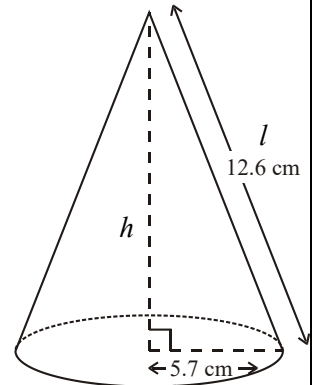
Calculate the volume of this pyramid.



2. Can you calculate the volume and surface area of a cone ?



- (a) Calculate the total surface area of the cone.
Give your answer correct to 3 sig. figures.



- (b) Calculate the volume of the cone.
Give your answer correct to 3 sig. figures.

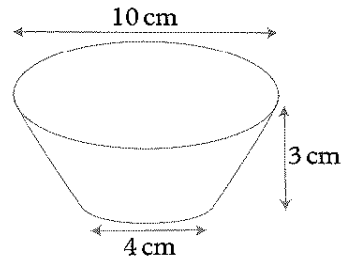
Hint: First find h!

3. Can you calculate the volume of frustum ?



A frustum is a cone with 'the end chopped off'. A bucket in the shape of a frustum as shown in the diagram has diameters of 10 cm and 4 cm at its ends and a depth of 3 cm.

Calculate the volume of the bucket.



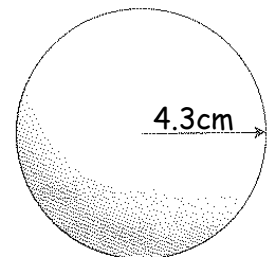
1. Sketch in big cone
2. Use similarity to find heights of big and small cones
3. Subtract small cone vol. from big cone vol. to find frustum vol. (using height to find cone volumes)

The bucket is made from 3mm thick plastic. Work out the total volume of plastic required to make one bucket.

4. Can you calculate the surface area and volume of a sphere ?



- a) Calculate the surface area of this sphere.
Give your answer to 3sf



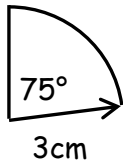
- b) Calculate the volume of this sphere.
Give your answer to 3sf

5. Can you find the area of a sector ?

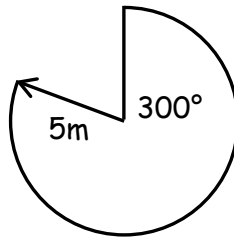


Find the area of these shapes. Round your answers to 3sf

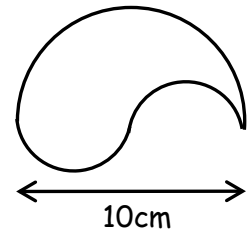
a)



b)



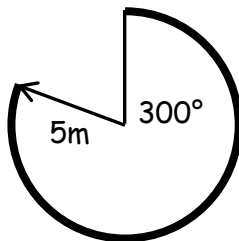
c)



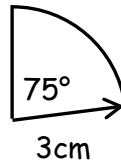
6. Can you find the length of an arc ?



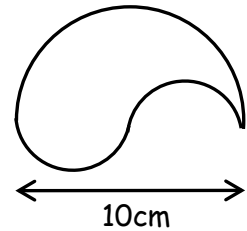
a) Find the length of the arc.
(to 3sf.)



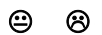
b) Find the perimeter
(to 3sf.)



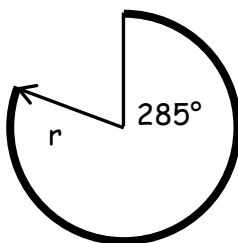
c) Find the perimeter.
(to 3sf.)



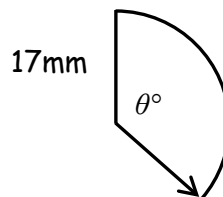
7. Can you work backwards to find the radius or sector angle ?



a) Find the radius (to 1dp) if the sector area is 200cm^2



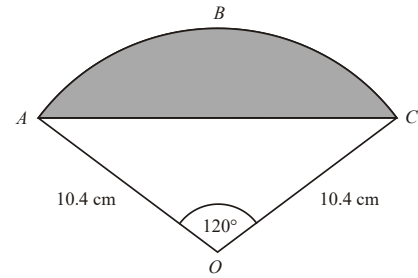
b) Find the sector angle (to 1dp) if the sector area is 250mm^2



8. Can you calculate the area of a circle segment ?



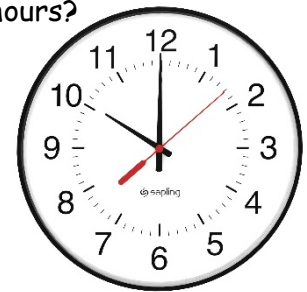
Calculate the area of the shaded region.
Round your answer to 3sf



9. Can you solve circle problems with a context ?



The minute hand of a clock is 12cm long. How far will it's tip travel in 2 hours?

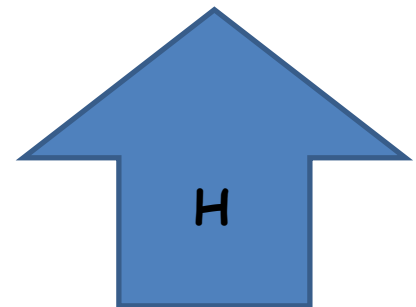


10. Can you solve problems about area and volume factors ?



Hint on calculating enlarged areas and volumes

<u>Object</u>		<u>Image</u>
Line (1 dimensional)	$\xrightarrow{\times SF^1}$	Enlarged Line
Area (2 dimensional)	$\xrightarrow{\times SF^2}$	Enlarged Area
Volume (3 dimensional)	$\xrightarrow{\times SF^3}$	Enlarged Volume

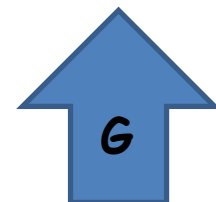


12cm

Shape H is an enlargement of Shape G.

The base of Shape G is 4cm. The base of Shape H is 12 cm

a) The height of H is 21cm, calculate the height of G.

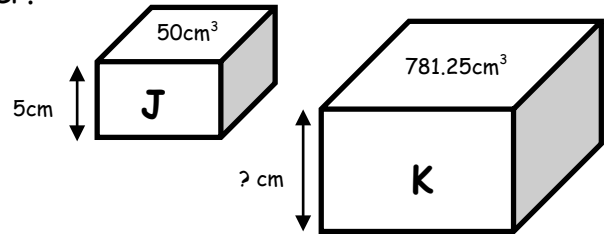


4cm

b) The area of G is 40cm^2 , calculate the area of H .

c) These cuboids are enlargements of each other.

- The height of J is 5cm
- The volume of J is 50cm^3
- The volume of K is 781.25cm^3
- Find the height of K



Data (FData2)

1. Can you complete histograms from frequency tables ?



2. Can you complete frequency tables from histograms ?



The histogram and frequency table show the weights of food eaten by Hemel the hamster each day for **26 days**. Use the histogram to complete the frequency table and use the frequency table to complete the histogram.

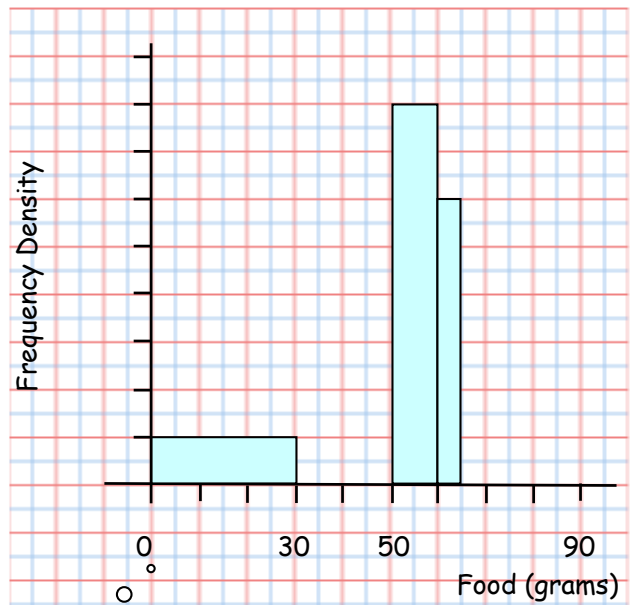
Food in grams	Freq	Width	Freq Density
$0 \leq w < 30$	3		
$30 \leq w < 50$			
$50 \leq w < 60$	8		
$60 \leq w < 65$			
$65 \leq w < 90$	5		

⋮

Hint: Total =

Hint: Complete FD scale first !

Food eaten by Hemel Hamster

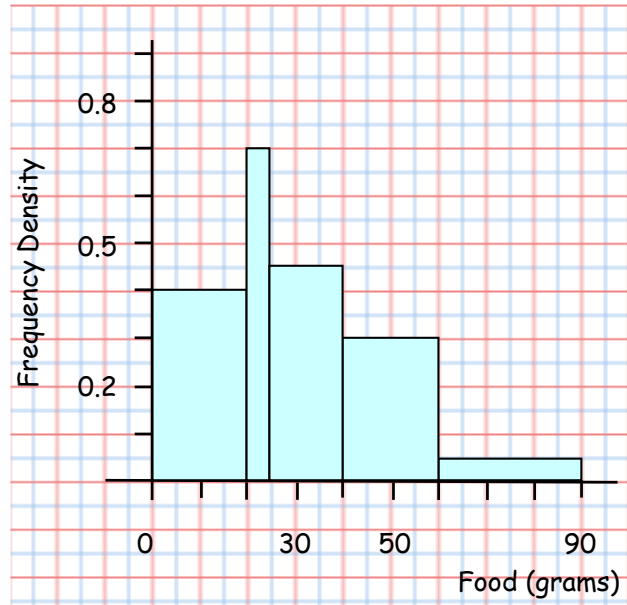


3. Can you draw and interpret frequency polygons ?



The food eaten over the same time period by another hamster, Hammy, is given in the histogram to the right.

Food eaten by Hammy Hamster



- Draw a frequency polygon over Hammy Hamster's histogram
- So you can compare the amount of food eaten by the two hamsters, draw in Hemel's frequency polygon on the same axes as Hammy's. (Use a different colour)
- Use your frequency polygons to compare the eating habits of the two hamsters. Ensure you refer to the skew of each data set.

4. Can you calculate the estimated mean from a histogram ?



- Calculate the estimated mean amount of food eaten per day by Hemel Hamster

Food in grams	Freq		

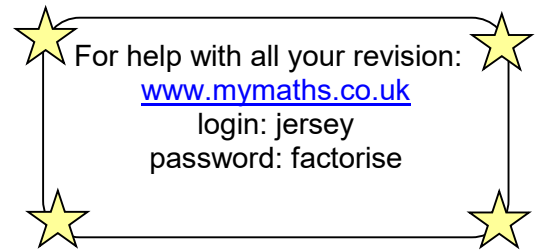
- Calculate the estimated mean amount of food eaten per day by Hammy Hamster

Food in grams	Freq		

Good Luck !

F3 ND Revision Worksheet

Number (ENum3)



1. Setting and solving proportional word problems



a) £10 gets you 12 euros, how many euros would you get for £17?

b) 4 men take 10 hours to paint a room. How long will it take 5 men?

c) 3 hairdressers cut 15 heads in 6 hours. How long will it take 2 hairdressers to cut 20 heads?

2. Can you solve direct and inverse proportion problems?



a) Q is directly proportional to R. When $Q = 72$, $R = 2$. Find the value of Q when $R = 10$.

b) N is inversely proportional to the square of P.
When $N = 4$, $P = 3$.
Find the value of N when $P = 14$.

c) The force, F newtons, exerted by a magnet on a metal object is inversely proportional to the square of the distance d cm. When $d = 2$ cm, $F = 50$ N.
(i) Express F in terms of d.
(ii) Find the force when the distance between the magnet and metal object is 10cm
(iii) Find the distance between the magnet and metal object when the force is 8N.
(iv) Explain what happens to F when d is halved.

